Краткие сообщения ОИЯИ №4[43]-90 *JINR Rapid Communications No.4[43]-90*
УДК 539.142

STRUCTURE OF HEAVY SKYRMIONS IN THE VARIATIONAL APPROACH

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The general structure of the solitons with baryon number 1-12 has been investigated in the framework of the variational approach to the Skyrme model. Some of the solitons have the toroidal structure and some of them are essentially more complicated. The masses of the obtained solitons are also given. These solitons could be interpreted as nuclear states after being quantized. A few of them could have isomer states which could differ by their form.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Структура тяжелых скирмионов в вариационном приближении

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Рассмотрена общая структура солитонов модели Скирма с барионными числами 1-12 в рамках вариационного приближения. Некоторые из солитонов имеют простую тороидальную структуру, другие же – существенно более сложную. Приводятся массы полученных солитонов. Будучи проквантованными, эти солитоны могли бы интерпретироваться как ядерные состояния. Некоторое число таких состояний может иметь изомерные состояния,отличающиеся формой распределения барионного заряда.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

1. Introduction

The Skyrme[1] model gives us straight way for constructing a system with an arbitrary baryon charge without using the so-called potential or adiabatic approach[2]. For this purpose we have to look for solitons of classical fields with the corresponding topological charge and then to quantize solitonic degrees of freedom to obtain an object with nuclear quantum numbers.

Up to now there are some papers concerning calculations of a quantum states with baryon number equal to 2,3 and 4 without vibrations[3] -[7] and including the breathing mode[8] -[9] in the Skyrme model. The most impressive results were obtained in the calculations of minimal-energy classical solitons for configurations with topological charge 2,3,4,5,6 by numerical methods [10].

In view of the fact that the minimum energy solutions up to $B = 6$ have been obtained by now without any limiting assumptions the variational approach can be useful when the constraints do not lead too far away from the true minimum configuration and add deeper insight to the nature of the true solution.

Recently a variational ansatz was proposed independently in [11],[12] (See, also[13]). The ansatz being very simple, gives the possibility to do analytical analysis of the nuclear problems. For example the describing of the nuclear compound-states including antibaryons in their structure are between them.

In this paper we present the results of our variational calculations of the classical soliton structure in the framework of the original $SU(2)$ Skyrme model for baryon number $B \leq 12$. After the quantization procedure these solitons are to be identified with nuclei.

2. Generalized Ansatz for the Static Solutions

Here we follow our paper[14] with some modifications. In variational form of the chiral field U:

$$U(\vec{r}) = cosF(r) + i(\vec{r} \cdot \vec{N})\ sinF(r), \tag{1}$$

we use more general assumption about the configuration of the isotopic vector field $\vec{N}$:

$$\vec{N} = \{cos(\Phi(\phi,\theta)) \cdot sin(T(\theta)),\ sin(\Phi(\phi,\theta)) \cdot sin(T(\theta)),\ cos(T(\theta))\}. \tag{2}$$

In eq.(2) $\Phi(\phi)$, $T(\theta)$ are some arbitrary functions of angles (θ,ϕ) of the vector $\vec{r}$ in the spherical coordinate system.

3. Mass Functional and Solutions for Static Equations

Let us consider the Lagrangian density $\mathcal{L}$ for the stationary solution:

$$\mathcal{L} = \frac{F_\pi^2}{16} \cdot Tr(L_k L_k) + \frac{1}{32e^2} \cdot Tr\left[L_k, L_i\right]^2. \tag{3}$$

Here $L_k = U^+\partial_k U$ are the left currents. After some tedious algebra, (1), (2), (3) lead to the expression

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4, \tag{4}$$

where

$$\mathcal{L}_2 = -\frac{F_\pi^2}{8}\left\{\left(\frac{\partial F}{\partial r}\right)^2 + \left[\left(\frac{sinT}{sin\theta}\frac{\partial\Phi}{\partial\phi}\right)^2 + \left(\frac{\partial T}{\partial\theta}\right)^2 + sin^2T\left(\frac{\partial\Phi}{\partial\theta}\right)^2\right]\frac{sin^2F}{r^2}\right\}$$

and

$$\begin{aligned}\mathcal{L}_4 &= -\frac{1}{2e^2}\cdot\frac{sin^2F}{r^2}\cdot\left\{\frac{sin^2T}{sin^2\theta}\left(\frac{\partial T}{\partial\theta}\right)^2\left(\frac{\partial\Phi}{\partial\phi}\right)^2\cdot\frac{sin^2F}{r^2}\right.\\ &+ \left.\left[\frac{sin^2T}{sin^2\theta}\left(\frac{\partial\Phi}{\partial\phi}\right)^2 + \left(\frac{\partial T}{\partial\theta}\right)^2 + sin^2T\left(\frac{\partial\Phi}{\partial\theta}\right)^2\right]\left(\frac{\partial F}{\partial r}\right)^2\right\}\cdot\end{aligned} \tag{5}$$

The variation of the functional $L = \int \mathcal{L}d\vec{r}$ with respect to Φ leads to an equation which has a solution of the type

$$\Phi(\theta,\phi) = k(\theta)\cdot\phi + Const \tag{6}$$

with a constraint:

$$\frac{\partial}{\partial\theta}\left[sin^2T(\theta)\cdot sin\theta\cdot\frac{\partial k(\theta)}{\partial\theta}\right] = 0. \tag{7}$$

It is easely seen from eqs.[14] that function $k(\theta)$ may be piecewise constant function (step function) in general case. Moreover $k(\theta)$ must be integer in any region $\theta_m \le \theta \le \theta_{m+1}$, where θ_m, θ_{m+1} are successive points of discontinuity. The positions of these are points determined by the condition

$$T(\theta_m) = m\cdot\pi, \tag{8}$$

with integer m, as follows from eq.(7).

Now we have the following expression for the mass of the soliton

$$M = \gamma\cdot[a\cdot A + b\cdot B + C], \tag{9}$$

where $\gamma = \pi\cdot F_\pi/e$ and $x = F_\pi\cdot e\cdot r$ and the a, b and A, B, C are the following integrals:

$$a=\int_0^\pi\Big[k^2(\theta)\frac{sin^2T}{sin^2\theta}+(T')^2\Big]sin\theta d\theta,\quad b=\int_0^\pi k^2(\theta)\frac{sin^2T}{sin^2\theta}(T')^2 sin\theta d\theta, \quad (10)$$

$$A=\int_0^\infty sin^2F\Big[\frac{1}{4}+(F')^2\Big]dx,\ B=\int_0^\infty\frac{sin^4F}{x^2}dx,\ C=\frac{1}{2}\int_0^\infty(F'x)^2dx. \quad (11)$$

Here we use the symbol prime to denote the following derivatives

$$T'=\frac{\partial T}{\partial\theta}\ ;\ F'=\frac{\partial F}{\partial x} \quad (12)$$

We consider the configurations with finite masses. The only configurations which obey the finiteness of mass condition are the configurations with $F(0)=n\cdot\pi$, where n-is some integer number. Without loss of generality we take $F(\infty)=0$. As it was shown in[14] $T(\theta)$ has the following behaviour near the boundary of the domain of its definition

$$T(\theta)\to\theta^k,\ for\ \theta\to 0\ ;\ T(\theta)\to\pi\cdot l-(\pi-\theta)^k,\ for\ \theta\to\pi. \quad (13)$$

Here l is an integer number. Thus we have the following estimation for the number of discontinuity points d:

$$0\le d\le l-1. \quad (14)$$

Now all solutions $U_{nl\{k(d)\}}$ are classified by a set of integer numbers n, l and $k_0,...,k_{l-1}$. The functions $F(x)$ and $T(\theta)$ have to obey the equations (14,15) from[14] in arbitrary space region with given number k.

4. Baryon Charge Distribution and the Soliton Structure

Now consider more carefully the structure of solitons. For that purpose let us calculate the baryon charge density

$$J_0^B(\vec{r})=-\frac{1}{24\pi^2}\cdot\epsilon_{0\mu\nu\rho}Tr(L_\mu L_\nu L_\rho). \quad (15)$$

The straightforward calculation gives

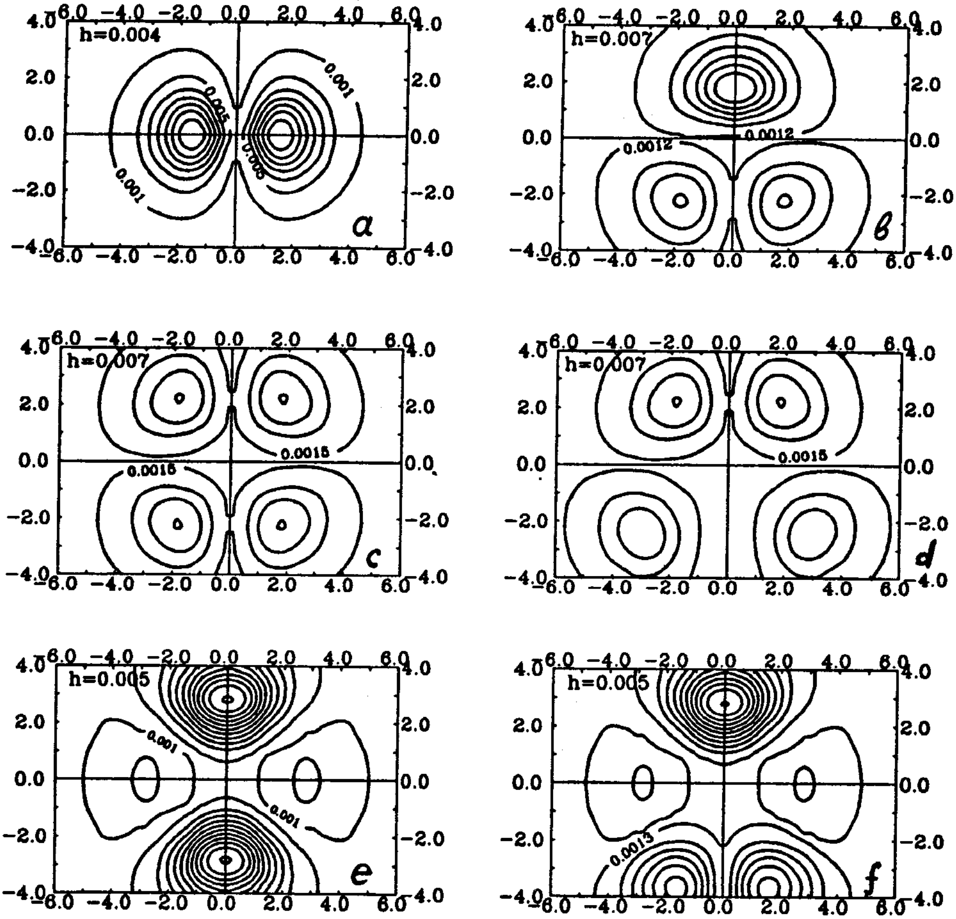
$$J_0^B(r,\theta)=-\frac{1}{2\pi^2}\cdot\frac{sin^2F}{r^2}\cdot\frac{dF}{dr}\cdot\frac{sinT}{sin\theta}\cdot\frac{dT}{d\theta}\cdot\frac{d\Phi}{d\phi}\cdot \quad (16)$$

Equation (16) immediately results in the expression for the corresponding topological charge

$$B = n \cdot \sum_{m=0}^{l-1} (-1)^m k_m \, . \tag{17}$$

In[14] we have investigated toroidal multiskyrmion configurations with baryon numbers $B = 1,2,3,4,5$ and more complicated nontoroidal (including antiskyrmions ($\bar{S}$)) configurations: $B = 0$ ($S - \bar{S}$, $2S - 2\bar{S}$, $3S - 3\bar{S}$), $B = 1$ ($S - \bar{S} - S$), $B = 2$ ($2S - 2\bar{S} - 2S$), $B = 3$ ($3S - 3\bar{S} - 3S$). As was pointed out in[14] the quantum states of the $S - \bar{S} - S$ type ($k = 1$, $l = 3$) should experimentally appear as compound nuclear states in the interaction of a stopped antiproton with a deutron. So we have unusual possibility to include antinucleons in the compound state structure in the same manner as nucleons.

The baryon charge distributions for the new solitons which we have obtained here are presented in Figs.1,2. In Figs.1,2 we present the baryon charge distribution in the (X, Z) plane for soliton with different topological structure and minimal possible value of the topological charge. The distribution presented in Fig.1(a) corresponds to the k-skyrmion (kS) configuration with $k = 2$. This configuration is the toroidal one with baryon charge $B = 2$, but $B = k$ with $k \geq 3$ of the same structure are also possible. In Fig.1(b) we see baryon charge distribution for the soliton with $B = 3$. This object consists of one toroid ($2S$) with $B = 2$ and one deformed soliton S (apple) with $B = 1$. The configuration of the same nature $S - kS$, $k \geq 2$ are possible for $B = |k| + 1$. Another possible configuration with $B = 3$ is given in Fig.1(e). It has the $S - kS - S$ structure with $k = 1$. The structure $kS - kS$ with $k \geq 2$ is presented in Fig.1(c) for the case $k = 2$. Such a structure containes configurations with baryon charges $B = 4,6,8,10$ and so on. The α-particle like object with $B = 4$ may also consist of one toroidal soliton with $B = 2$ and two solitons with $B = 1$ (it is like 1(e)-figure). One toroidal soliton with $B = 3$ and one nontoroidal soliton (apple) with $B = 1$ is the third possibility to construct the object with total $B = 4$ (like configuration presented in Fig.1(b)). We have to note that $B = 4$ soliton may also consist of one toroidal soliton with $B = 2$, one toroidal soliton with $B = 1$ and one apple-soliton with $B = 1$ but its mass is essentially higher than the mass of more symmetric structure, described above. The configurations which are like one presented in Fig.1(f) may also have the following structure: $S - kS - k'S$ ($k \geq 1$, $B = 1 + |k| + |k'|$). The examples of the general $k_1 S - k_2 S - \ldots - k_n S$ -type structure with $B = 4,5,6$ are given in Figs.2(a-



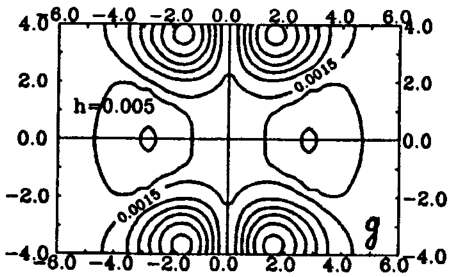


Fig.1. Baryonic number density for skyrmion with given B (z and x are coordinates scaled by eF_π, h is the contour interval): a) B = 2, (S-S); b) B = 3, (S-2S); c) B = 4, (2S-2S); d) B = 5, (2S-3S); e) B = 3, (S-S-S); f) B = 4, (S-S-2S); g) B = 5, (2S-S-2S).

c). We note that the deformed toroidal structure with $B = 1$ may only be placed in internal regions of the structure and the apple-soliton with $B = 1$ may be placed only in external regions. Moreover the toroid with greater baryon charge B has greater radius.

5. The Masses of Classical Solitons

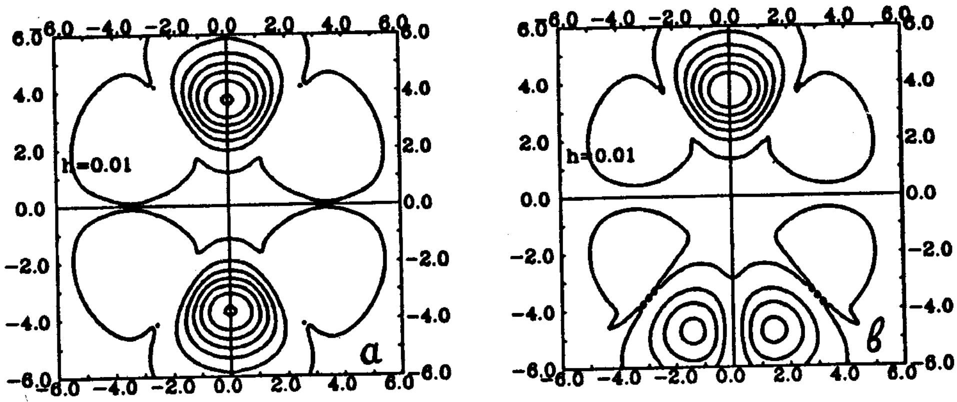
When we discuss multiskyrmion configurations we search for and investigate not only classically stable configurations (The decay in two or more skyrmions is forbidden energetically). Nonstable configurations are also in our attention because they may become stable after the quantization procedure [14].

In Table1 we present the masses of the states consisting of only one toroidal soliton[14]. These solitons have baryon charge $B = k$ and $l = 1$. The calculated soliton masses are given in $(\pi F_\pi/e)$ units.

Table1. The classical masses of toroidal solitons

B	1	2	3	4	5
M	11.605	22.458	34.585	47.675	61.569

From Table 1one sees almost linear dependence of the classical masses



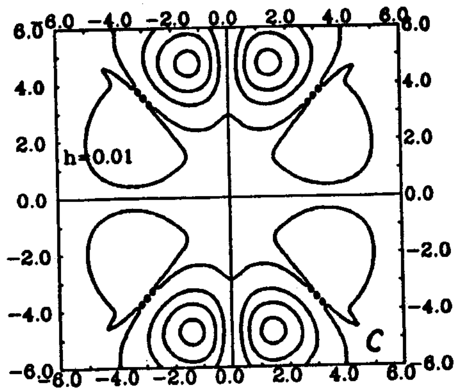


Fig.2. The same as in Fig.1: a) B = 4, (S-S-S-S); b) B = 5, (S-S-S-2S); c) B = 6, (2S-S-S-2S).

on the baryon charge. Such a dependence strongly differs from $M \sim B(B+1)$ that may be obtained for the hedgehog ansatz[15].

We have to look for possible nontoroidal solitons for two reasons. First, there is a number of solutions with smaller masses among the nontoroidal solitons. For example a classical soliton composed from two toroidal dibaryons has a mass smaller than the mass presented in Table 1 and corresponding to same baryon charge $B = 4$. Now this state cannot decay into four skyrmions with $B = 1$ in contrast with the case of one toroidal skyrmion with $B = 4$. Second - there are no states with triton ground state quantum numbers among quantum states of the toroidal skyrmion as one can see from [14]. On the other hand,the configuration presented in Fig.1(a) has the mass very near to the mass of the toroidal skyrmion with the same baryon number. The corresponding to this configuration quantum state may have chance to have "right" quantum numbers and to be bounded after quantization.

In Table 2 are presented the lowest masses of skyrmions with $1 \leq B \leq 12$.

Table2. The lowest masses of the solitons for topological sectors with $B \leq 12$.

B	1	2	3	4	5
M	11.605	22.458	34.585	45.536	56.118
B	6	8	9	10	12
M	66.701	89.310	103.08	113.12	134.45

According to our calculations the masses of the solitons with equal baryon charges strongly differ if they have different structure. For example the masses of solitons with $B = 5$ are $M_1 = 65.35$, $M_2 = 69.14$ and $M_3 = 56.12$. Their baryon charge distributions are like ones presented in Fig.1(f,e,d) and show very different structures. The soliton with the lowest mass consists of two toroids with baryon number $B = 2$ and $B = 3$.

6. Conclusions

The classical solitons with baryon number $B \leq 12$ have been investigated in the framework of the very general assumption about the form of the solution of the Skyrme model equations. The calculated solitons have very complicated structure and only some of them have simple toroidal structure. These solitons could be interpreted as nuclear physical states after being quantized. Some of them could have isomer states. Such isomers could differ by their form. In conclusion we have to note that our ansatz leads to stable solitons with even baryon charge as well as to solitons with odd ones.

Acknowledgements. We thank prof. V.G. Kadyshevsky and Drs. V.Yu. Novozhilov, M.K. Volkov, D.I. Kazakov for very useful discussions of our results.

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Received on June 11, 1990.